

35-AMALIY MASHG'ULOT

Ish vaqti va bosqichlari	O'qituvchi faoliyatining mazmuni	Talabalar faoliyatining mazmuni
I-Bosqich O'quv mashg'ulotiga kirish (10 daqiqa)	1.1. Mashg'ulot mavzusi bo'yicha tushuncha berish. 1.2. Mavzuning avvalgi o'tilgan mavzu bilan aloqadorligi bo'yicha tushuncha berish	Tinglaydilar, yozadilar
II-Bosqich Asosiy (65 daqiqa)	2.1. Amaliy mashg'ulot mavzusiga oid nazariy ma'lumot berish. 2.2. Mavzuga oid amaliy topshiriqlar bajarish.	Tinglaydilar, yozadilar, savollar beradi, bajaradilar.
III-Bosqich Yakunlovchi (5 daqiqa)	3.1. Amaliy mashg'ulotni yakunlash 3.2. Talabalarga mustaqil bajarish uchun topshiriqlar berish.	Tinglaydilar, yozadilar, savollar beradi, bajaradilar.

35-MAVZU. BIR JINSLIMAS DIFFERENTIAL TENGLAMALAR

1-reja. Bizga chiziqli bir jinsli tenglama berilgan:

$$y^{(n)} + p_1(x)y^{(n-1)} + \dots + p_{n-1}(x)y' + p_n(x)y = 0. \quad (1)$$

Erkli o'zgaruvchini almashtirish bilan bu tenglamani o'zgarmas koeffisientli chiziqli tenglamaga olib kelish masalasi bilan shug'ullanamiz. $t = u(x)$ almashtirish bajaraylik. U holda:

$$y' = y'_t t'_x = y'_t u'(x)$$

$$y'' = y''_{t^2} [u'(x)]^2 + y'_t u''(x)$$

.....

$$y^{(n)} = y^{(n)}_{t^{(n)}} [u'(x)]^n + \dots + y'_t u^{(n)}(x)$$

Bu hisoblashlar ko'rsatadiki yuqoridagi almashtirishdan keyin (1) tenglama $y^{(n)}_{t^{(n)}} [u'(x)]^n + \dots + p_n(x)y = 0$ ko'rinishni oladi. Buni $[u'(x)]^n$ ga bo'lamiz:

$$y^{(n)}_{t^{(n)}} + \dots + \frac{p_n(x)}{[u'(x)]^n} y = 0. \text{ Bu yerda } u(x) \text{ funksiyani shunday tanlash kerakki } y$$

oldidagi koeffisient o'zgarmas songa aylansin. $\frac{p_n(x)}{[u'(x)]^n} = \frac{1}{c^n}$ desak, u holda

$u'(x) = c \sqrt[n]{p_n(x)}$ yoki $u(x) = c \int \sqrt[n]{p_n(x)} dx$. Shunday qilib, agar (1) tenglama erkli o'zgaruvchini almashtirish bilan o'zgarmas koeffisientli tenglamaga aylansa u holda almashtirish fomulasi

$$t = c \int \sqrt[n]{p_n(x)} dx \quad (2)$$

ko'rinishda bo'lishi zarur.

2-reja. Endi **Eylarning chiziqli tenglamasini** qaraymiz:

$$D(y) \equiv x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + \dots + a_{n-1} x y' + a_n y = 0, \quad (3)$$

bu yerda $a_1, a_2, \dots, a_n$ - o'zgarmas haqiqiy sonlar. Bu tenglamani (1) tenglama bilan taqqoslab $p_n(x) = \frac{a_n}{x^n}$ ekanligini ko'ramiz. (2)ni hisobga olib (3) tenglamada

$t = c \int \sqrt[n]{\frac{a_n}{x^n}} dx$ yoki $c = \frac{1}{\sqrt[n]{a_n}}$ deb olib, $t = \ln x$ almashtirish bajaramiz. U holda:

$$y' = y'_t \cdot \frac{1}{x}$$

$$y'' = y''_{t^2} \frac{1}{x^2} - y'_t \frac{1}{x^2}$$

$$y''' = y'''_{t^3} \frac{1}{x^3} - 3y''_{t^2} \frac{1}{x^3} + 2y'_t \frac{1}{x^3}$$

.....

$$y^{(n)} = y^{(n)}_{t^n} \frac{1}{x^n} + \dots + (-1)(n-1)! y'_t \frac{1}{x^n}$$

Bu hisoblashlar ko'rsatadiki $x^i y^{(i)}$ ifoda $y'_t, y''_{t^2}, \dots, y^{(i)}_{t^i}$ larning chiziqli kombinatsiyasidan iborat va (3) tenglamada ularni mos ravishda qo'ysak o'zgarmas koeffisientli tenglama hosil bo'ladi.

1-Misol. $x^2 y'' - 2xy' + 2y = 0$ tenglamaning umumiy yechimini topamiz.

$t = \ln x$ almashtirish bajarsak: $y' = y'_t \cdot \frac{1}{x}$, $y'' = y''_{t^2} \frac{1}{x^2} - y'_t \frac{1}{x^2}$. Bularni berilgan tenglamaga qo'yamiz: $y''_{t^2} - y'_t - 2y'_t + 2y = 0$ yoki $y''_{t^2} - 3y'_t + 2y = 0$. Bu o'zgarmas koeffisientli tenglamaning harakteristik tenglamasi: $\lambda^2 - 3\lambda + 2 = 0$. $\lambda_1 = 1$, $\lambda_2 = 2$. Demak $y = C_1 e^t + C_2 e^{2t}$. Almashtirish formulasi bo'yicha x o'zgaruvchini qaytaramiz: $y = C_1 x + C_2 x^2$. **Javob:** $y = C_1 x + C_2 x^2$.

3-reja. Chebishev tenglamasini qaraymiz:

$$(1-x^2)y'' - xy' + n^2 y = 0 \quad (5)$$

Bu tenglama $(-\infty, -1)$, $(-1, +1)$, $(1, +\infty)$ intervallarning har birida mavjudlik va yagonalik teoremlarini qanoatlantiradi. Biz (5) tenglamaning $(-1, +1)$ intervaldagi umumiy yechimini quramiz. (2) formulaga ko'ra quyidagiga egamiz:

$$t = c \int \sqrt{\frac{n^2}{1-x^2}} dx$$

Bu yerda $c = -\frac{1}{n}$ deb olsak $t = \arccos x$ yoki $x = \cos t$ almashtirish formulasi hosil bo'ladi. Bundan

$$y' = y'_t t'_x = y'_t \frac{1}{x'_t} = -y'_t \frac{1}{\sin t}$$

$$y'' = y''_{t^2} \frac{1}{\sin^2 t} - y'_t \frac{\cos t}{\sin^3 t}$$

Bularni (5) tenglamaga qo'ysak $y''_{t^2} - y'_t \frac{\cos t}{\sin t} + y'_t \frac{\cos t}{\sin t} + n^2 y = 0$ yoki

$$y''_{t^2} + n^2 y = 0$$

o'zgarmas koeffisientli tenglamani hosil qilamiz. Uning umumiy yechimi $y = C_1 \cos nt + C_2 \sin nt$ formulaga ega. Bundan **Chebishev tenglamasining umumiy yechimini** hosil qilamiz: $y = C_1 \cos n \arccos x + C_2 \sin n \arccos x$.

Misollar va masalalar

Tenglamani umumiy yechimini toping:

$$1. y'' - 2y' + y = \frac{e^x}{\sqrt{4-x^2}}.$$

$$\mathcal{K}: y = (C_1 + \sqrt{4-x^2} + \arcsin \frac{x}{2} + C_2 x) e^x.$$

$$2. y'' + y = \frac{1}{\sqrt{\cos 2x}}.$$

$$\mathcal{K}: y = C_1 \cos x + C_2 \sin x + \frac{1}{\sqrt{2}} \cos x \ln \left| \cos x + \sqrt{\cos^2 x - \frac{1}{2}} \right| + \frac{1}{\sqrt{2}} \sin x \cdot \arcsin(\sqrt{2} \sin x).$$

$$3: y'' - y' = -e^{2x} \sqrt{1-e^{2x}}.$$

$$\mathcal{K}: y = C_1 + C_2 e^x + \frac{1}{2} e^x (\arcsin e^x + e^x \sqrt{1-e^{2x}}) + \frac{1}{3} \sqrt{(1-e^{2x})^3}.$$

Koshi masalasini yeching:

$$y'' + 4y = \frac{1}{\sin^2 x}; \quad y \Big|_{x=\frac{\pi}{2}} = 0; \quad y' \Big|_{x=\frac{\pi}{2}} = \pi.$$

$$\mathcal{K}: y = -\cos 2x \ln |\sin x| - x \sin 2x - \cos^2 x.$$